

SCHOOL: Your school name

SENIOR THREE MATHEMATICS · UNIT 10 HOMEWORK

Collinear points and orthogonal vectors in 2D · FREE SAMPLE

MARKING SCHEME (TEACHER'S COPY)

Academic year: 20..... – 20.....

Date:/...../20.....

Duration: Take-home

Total marks: 18

INSTRUCTIONS

1. Answer ALL questions.
2. Show all your working.
3. This free sample is a short extract. A full homework sheet has 25 marks.

PART 1: QUICK QUESTIONS

Write the letter of the correct answer (1 mark each).

1. Find k if $\begin{pmatrix} k \\ 2 \end{pmatrix} \perp \begin{pmatrix} 3 \\ 6 \end{pmatrix}$. (1 mark)
- A. -1
 B. 4
 C. 1
 D. -4

Answer: D. -4

Explanation: $3k + 12 = 0$.

2. Which vector is orthogonal to $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$? (1 mark)
- A. $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$
 B. $\begin{pmatrix} 10 \\ 4 \end{pmatrix}$
 C. $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$
 D. $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$

Answer: D. $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$

Explanation: $-10 + 10 = 0$.

3. Find $\begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -1 \end{pmatrix}$. (1 mark)
- A. $\begin{pmatrix} 8 \\ -3 \end{pmatrix}$
 B. 5
 C. 14
 D. 11

Answer: B. 5

Explanation: $8 - 3 = 5$.

4. Are $A(0,0)$, $B(1,2)$, $C(3,6)$ collinear? (1 mark)
- A. Only A and B
 B. Cannot tell
 C. No
 D. Yes

Answer: D. Yes

Explanation: $\overrightarrow{AC} = 3\overrightarrow{AB}$.

PART 2: SHORT ANSWERS

Show your working.

5. Find k such that $A(1,k)$, $B(3,5)$, $C(7,13)$ are collinear. (2 marks)

Model answer: $\overrightarrow{BC} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$, gradient 2; $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 5-k \end{pmatrix}$ must have $5-k=4$, $k=1$.

Marking: 2 marks: correct answer with the working or reason.

Common mistake: Using a perpendicular condition.

6. Show that $P(-1,-3)$, $Q(1,1)$ and $R(3,5)$ are collinear. (3 marks)

Model answer: $\overrightarrow{PQ} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$, $\overrightarrow{PR} = \begin{pmatrix} 4 \\ 8 \end{pmatrix} = 2\overrightarrow{PQ}$. Parallel with common point P : collinear.

Marking: Method: 2 marks for the working (see the steps above) · final answer: 1 mark. Accept any correct method.

Common mistake: Forgetting to mention the common point.

7. Find a unit vector orthogonal to $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$. (2 marks)

Answer: $\begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix}$

Model answer: $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ is orthogonal; divide by its length 5.

Marking: Method: 1 mark for the working (see the steps above) · final answer: 1 mark.

Accept any correct method.

PART 3: STRUCTURED PROBLEM

Show all your working.

8. Collinear points and orthogonal vectors in 2D

(7 marks)

- (a) Find k so that $\begin{pmatrix} 3 \\ k \end{pmatrix} \perp \begin{pmatrix} 4 \\ -6 \end{pmatrix}$.

(3 marks)

Model answer: $12 - 6k = 0$, so $k = 2$.

Marking: Method: 2 marks for the working (see the steps above) · final answer: 1 mark.
Accept any correct method.

- (b) Find k so that $A(1, 1)$, $B(3, k)$ and $C(5, 9)$ are collinear.

(4 marks)

Model answer: $\overrightarrow{AB} = \begin{pmatrix} 2 \\ k-1 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$.

Determinant: $2 \times 8 - 4(k-1) = 0$.

$16 = 4k - 4$, so $k = 5$.

Marking: Method: 3 marks for the working (see the steps above) · final answer: 1 mark.
Accept any correct method.

Common mistake: The determinant is $ad - bc$ with the components crossed correctly.

END