

SCHOOL: **Your school name****SENIOR FOUR MATHEMATICS · UNIT 9 QUIZ****Vector space of real numbers · FREE SAMPLE****MARKING SCHEME (TEACHER'S COPY)****Academic year:** 20..... – 20.....**Date:**/...../20.....**Duration:** 1 hour 30 minutes**Total marks:** 24**INSTRUCTIONS**

1. A full unit quiz has three sections and 50 marks.
2. This free sample is a short extract of each section.
3. Calculators may be used unless a question says otherwise.

SECTION A: OBJECTIVE QUESTIONS*Answer ALL questions. Each question carries 1 mark.*

1. Do $(1, 2)$ and $(3, 5)$ form a basis of $\mathbb{R}^2$? (1 mark)
- A. Yes: the determinant is $-1 \neq 0$
 - B. No: the determinant is 0
 - C. Yes: both vectors are non-zero
 - D. No: a basis needs perpendicular vectors

Answer: A. Yes: the determinant is $-1 \neq 0$ **Explanation:** Step 1. $\det = 1 \times 5 - 3 \times 2 = -1$.Step 2. Non-zero: independent; two independent vectors of $\mathbb{R}^2$ form a basis.

2. Find the area of the parallelogram built on $(3, 1)$ and $(1, 2)$. (1 mark)
- A. $\sqrt{50}$
 - B. 6
 - C. 7
 - D. 5

Answer: D. 5**Explanation:** Step 1. Area = $|\det| = |3 \times 2 - 1 \times 1|$.Step 2. $|6 - 1| = 5$.

3. Find k so that $(k, 3)$ and $(2, 6)$ are linearly dependent. (1 mark)
- A. $k = -1$
 - B. $k = 1$
 - C. $k = 4$
 - D. $k = 9$

Answer: B. $k = 1$ **Explanation:** Step 1. Determinant: $6k - 3 \times 2 = 0$.Step 2. $k = 1$ (indeed $(2, 6) = 2(1, 3)$).

4. Find the coordinates of $(5, 8)$ in the basis $\{(1, 2), (3, 5)\}$. (1 mark)
- A. $(1, 1)$
 B. $(-1, 2)$
 C. $(5, 8)$
 D. $(2, -1)$

Answer: B. $(-1, 2)$

Explanation: Step 1. $a + 3b = 5$ and $2a + 5b = 8$.

Step 2. From the first: $a = 5 - 3b$; then $10 - 6b + 5b = 8$, so $b = 2$, $a = -1$.

Step 3. Check: $-(1, 2) + 2(3, 5) = (5, 8)$. ✓

SECTION B: SHORT ANSWER QUESTIONS

Answer ALL questions. Show your working.

5. Find the angle between $\vec{u} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$. (4 marks)

Model answer: $\vec{u} \cdot \vec{v} = 3 + 2 = 5$; $|\vec{u}| = \sqrt{5}$, $|\vec{v}| = \sqrt{10}$.

$$\cos \theta = \frac{5}{\sqrt{50}} = \frac{1}{\sqrt{2}}.$$

$$\theta = 45^\circ.$$

Marking: Method: 3 marks for the working (see the steps above) · final answer: 1 mark.
 Accept any correct method.

Common mistake: Divide by the product of the **magnitudes**, not by the sum.

6. Are $\vec{u} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$ linearly independent? (3 marks)

Model answer: Determinant: $2 \times 6 - 4 \times 3 = 0$.

So they are dependent: $\vec{v} = 2\vec{u}$.

Marking: Method: 2 marks for the working (see the steps above) · final answer: 1 mark.
 Accept any correct method.

Common mistake: Determinant zero means dependent; non-zero means independent.

7. Give a basis of $W = \{(x, y) : x + 4y = 0\}$. (3 marks)

Answer: $\{(-4, 1)\}$

Model answer: Step 1. $x = -4y$, so $(x, y) = y(-4, 1)$.

Step 2. One vector spans W : dimension 1.

Marking: Method: 2 marks for the working (see the steps above) · final answer: 1 mark.
 Accept any correct method.

8. Find $\vec{u} \cdot \vec{v}$ for $\vec{u} = (2, -3)$ and $\vec{v} = (4, 1)$. (3 marks)

Answer: 5

Model answer: Step 1. $2 \times 4 + (-3) \times 1$.

Step 2. $8 - 3 = 5$.

Marking: Method: 2 marks for the working (see the steps above) · final answer: 1 mark.
Accept any correct method.

SECTION C: STRUCTURED QUESTIONS

Answer ALL questions. Show all your working.

9. Vector space of real numbers

(7 marks)

(a) Show that $W = \{(x, y) : x + 2y = 0\}$ is a subspace of $\mathbb{R}^2$ and give a basis. (4 marks)

Model answer: $(0, 0) \in W$ [1]

Closed under addition [1]

Closed under scalar multiplication [1]

Basis $\left\{ \begin{pmatrix} -2 \\ 1 \end{pmatrix} \right\}$ [1]

Marking: Give the marks shown in brackets [] for each step. Accept any correct equivalent method.

(b) $\vec{u} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$. Find $|\vec{u}|$, the unit vector in its direction, and $2\vec{u} - \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. (3 marks)

Model answer: $|\vec{u}| = 5$ [1]

$\begin{pmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix}$ [1]

$\begin{pmatrix} 5 \\ -9 \end{pmatrix}$ [1]

Marking: Give the marks shown in brackets [] for each step. Accept any correct equivalent method.

END